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STABILITY OF A VISCOELASTIC ROD ON DYNAMIC LOADING

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There are fairly numerous papers on the dynamic loading in the elastic range for rods, which have been surveyed in [1, 2]; however, comparatively little is known about the dynamic stability of rods on the viscoelastic range.

Here we show that one can examine the dynamic stability of such a rod under increasing compression from nonlinear integrodifferential equations containing variable coefficients, which can be solved numerically by means of quadratures. We consider how the major factors affect the behavior.

1. Many aspects of nonlinear oscillations and dynamic stability can be considered by means of nonlinear integrodifferential equations with variable coefficients [3-7] for rods and beams composed of composite materials with viscoelastic behavior:

$$\ddot{T}_k+\omega_k^2[1-\mu_k P(t)]\,T_k=X_k\left\{t,\ T_1,\ \ldots,\ T_N,\ \int_0^t \varphi_k(t,\tau,T_1(\tau),\ \ldots,\ T_N(\tau))d\tau\right\},$$
$$T_k(0)=T_{0k},\ \dot{T}_k(0)=\dot{T}_{0k},\ k=1,\ldots,N, \tag{1.1}$$

in which $T_k = T_k(t)$ are time functions to be determined, P, X_k, and φ_k are given continuous functions in the argument range, and ω_k =, μ_k = const.

A numerical method has been proposed [8, 9] based on the quadrature formulas for integrodifferential equations; here that method is extended to (1.1), for which the system is written in integral form. We put $t = t_m$, $t_m = mh$ (h = const, m = 1, 2,...) and replace the integrals by certain quadrature formulas to get a recurrent formula for $T_{mk} = T_k(t_m)$:

$$T_{mk}=T_{0k}\cos\omega_k t_m+\frac{\dot{T}_{0k}}{\omega_k}\sin\omega_k t_m+\frac{1}{\omega_k}\sum_{r=0}^{m-1}A_r^{(k)}\Big\{\mu_k\omega_k^2P_rT_{rk}+$$
$$+X_k\Big(t_r,\ T_{r1},\ \ldots,\ T_{rN},\ \sum_{s=0}^{r}B_s^{(k)}\varphi_k(t_r,t_s,T_{s1},\ \ldots,\ T_{sN})\Big)\Big\}\sin\omega_k(t_m-t_r),$$
$$m=1,2,\ \ldots,\ k=1,\ \ldots,\ N \tag{1.2}$$

in which $A_r^{(k)}$, $B_s^{(k)}$ are numerical coefficients independent of the form of the integrand functions and which take various values in accordance with the quadrature formulas [10].

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The basis for this method has been given in [11]; the error of the method is that obtained from the use of the quadrature formulas and has the same order of smallness with respect to the interpolation step.

2. We consider a viscoelastic rod hinge-attached at the ends and subject to a compressive force P that varies in time t: P = P(t). We assume that the rod has an initial deflection $u_0 = u_0(x)$ and that the cross section is constant over the length.

The relation between the stress σ and strain ε is taken as

$$\sigma = E(1 - R^*)(\varepsilon + \gamma\varepsilon^3), \quad R^*\varphi = \int_0^t R(t-\tau)\varphi(\tau)\,d\tau,$$

in which E is the instantaneous elastic modulus, R(t) the relaxation kernel, and γ the nonlinearity coefficient, which is dependent on the physical properties of the material.

We take the strain on the Bernoulli–Euler assumption as $\varepsilon = -z\partial^2(u - u_0)/\partial x^2$ [$u = u(x, t)$, this being the total transverse deflection, and z the distance from a point in the cross section to the neutral axis].

With these assumptions, the differential equation for the curved axis of the rod is [1, 4]

$$EJ(1 - R^*)\frac{\partial^4(u - u_0)}{\partial x^4} + P(t)\frac{\partial^2 u}{\partial x^2} + m\frac{\partial^2 u}{\partial t^2} = f - 3\gamma EJ_1(1 - R^*)\times$$
$$\times\left[2\frac{\partial^2(u - u_0)}{\partial x^2}\left(\frac{\partial^3(u - u_0)}{\partial x^3}\right)^2 + \left(\frac{\partial^2(u - u_0)}{\partial x^2}\right)^2\frac{\partial^4(u - u_0)}{\partial x^4}\right], \tag{2.1}$$

in which EJ is the bending rigidity, m the mass per unit length, $J_1 = \int_F z^4 dF$, F the cross-sectional area, and f the additional static load.

We write the solution to (2.1) satisfying the boundary conditions as

$$u(x, t) = \sum_{k=1}^{N} T_k(t)\sin\frac{k\pi x}{l}, \quad u_0(x) = \sum_{k=1}^{N} T_{0k}\sin\frac{k\pi x}{l} \tag{2.2}$$

in which l is the rod length; we substitute (2.2) into (2.1) and perform the Bubnov–Galerkin procedure to get a system of nonlinear integrodifferential equations for $T_k = T_k(t)$:

$$\ddot{T}_k + k^2\omega^2\left[k^2(1 - R^*) - \frac{P(t)}{P_e}\right]T_k = k^4\omega^2(1 - R^*)T_{0k} + \frac{4\alpha_k f}{mk\pi} -$$
$$-\frac{3\gamma\omega^2 J_1}{4J}\left(\frac{\pi}{l}\right)^4\sum_{n,i,j=1}^{N} a_{knij}(1 - R^*)(T_n - T_{0n})(T_i - T_{0i})(T_j - T_{0j}), \quad k = 1, \ldots, N. \tag{2.3}$$

Here P_e is the Euler critical load, $\omega = \sqrt{\frac{EJ}{m}\left(\frac{\pi}{l}\right)^4}$ is the frequency of the fundamental oscillation of the rod, and α_k is 1 if k is odd and 0 if k is even;

$$a_{knij} = n^2i^2j^2[-2ij(\delta_{n-k+i+j} + \delta_{n-k-i-j} + \delta_{n-k+i-j} +$$
$$+ \delta_{n-k-i+j} - \delta_{n+k-i-j} - \delta_{n+k+i-j} - \delta_{n+k-i+j}) + j^2(\delta_{n-k+j-k} + \delta_{n-i-j+k} -$$
$$- \delta_{n-i+j+k} - \delta_{n-i-j-k} - \delta_{n+i+j-k} - \delta_{n+i-j+k} + \delta_{n+i-j-k})]; \; \delta_i = \begin{cases}1 & \text{for } i = 0;\\ 0 & \text{for } i \neq 0.\end{cases}$$

We consider the case where P(t) increases in proportion to time; let P(t) = cFt, in which c is the rate of change. We introduce dimensionless quantities into (2.3):

$$\frac{T_k}{i}, \; \frac{T_{0k}}{i}, \; t^* = \frac{\omega t}{\sqrt{S^*}} = \frac{P}{P_e}, \; \frac{\sqrt{S^*}}{\omega}R(t), \; \frac{f}{i\omega^2 m}, \; \frac{3\gamma J_1 i^2}{4J}\left(\frac{\pi}{l}\right)^4$$

and retain the previous symbols to get

$$\frac{1}{S^*}\ddot{T}_k - k^2[t^* - k^2(1 - R^*)]T_k = k^4(1 - R^*)T_{0k} + \frac{4\alpha_k}{k\pi}f -$$
$$-\gamma\sum_{n,i,j=1}^{N} a_{knij}(1 - R^*)(T_n - T_{0n})(T_i - T_{0i})(T_j - T_{0j}), \tag{2.4}$$
$$T_k(0) = T_{0k}, \; \dot{T}_k(0) = \dot{T}_{0k}, \; k = 1, \ldots, N.$$

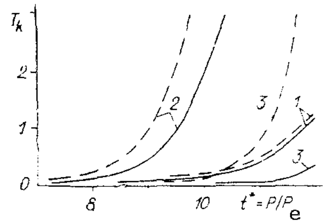

Fig. 1

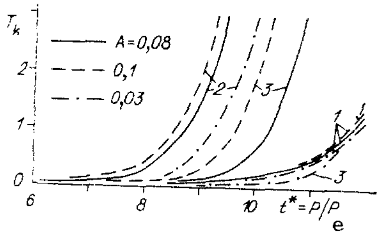

Fig. 2

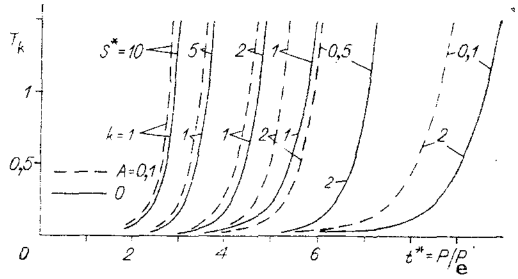

Fig. 3

Here $i = \sqrt{J/F}$ is the radius of inertia of the cross section, $S^* = P_e^{*3}(\pi vE/cl)^2$ the dimensionless loading rate parameter, $P_e^* = P_e/EF$ the dimensionless Euler load parameter, $v = \sqrt{E/\rho}$ the speed of sound in the material, and ρ the density.

We thus have a Cauchy problem for the dimensionless unknowns T_k, k = 1,...N; (2.4) is a particular case of (1.1). A numerical method is used to integrate (2.4) within wide ranges for the mechanical parameters as proposed in Sec. 1; (1.2) with the Koltunov–Rzhanitsyn kernel $R(t) = At^{\alpha-1}\exp(-\beta t)$, $\alpha = 0.25$, $\beta = 0.05$ takes the form

$$\begin{aligned} T_{mk} &= T_{0k}\cos\lambda_k t_m + \frac{\dot{T}_{0k}}{\lambda_k}\sin\lambda_k t_m + \frac{S^*}{\lambda_k}\sum_{r=0}^{m-1} A_r\Bigg\{\frac{4\alpha_k f}{k\pi} + k^2 t_r T_{rk} - \\ &- \gamma\sum_{n,i,j=1}^{N} a_{knij}(T_{rn}-T_{0n})(T_{ri}-T_{0i})(T_{rj}-T_{0j}) + \frac{k^4 A}{\alpha}\sum_{s=0}^{r} B_s(T_{r-s,k}-T_{0k})\times \\ &\times\exp(-\beta t_s) + \frac{\gamma A}{\alpha}\sum_{n,i,j=1}^{N} a_{knij}\sum_{s=0}^{r} B_s(T_{r-s,n}-T_{0n})(T_{r-s,i}-T_{0i})(T_{r-s,j}-T_{0j})\times \\ &\times\exp(-\beta t_s)\Bigg\}\sin\lambda_k(t_m-t_r), \quad k=1,\ldots,N, \quad m=1,2,\ldots, \end{aligned} \tag{2.5}$$

in which

$$\lambda_k = \sqrt{S^*}k^2;\ A_0 = h/2,\ A_r = h,\ r = 1,\ \ldots,\ m-1;\ B_0 = h^\alpha/2,\ B_r = \\ = h^\alpha[r^\alpha - (r-1)^\alpha]/2,\ B_s = h^\alpha[(s+1)^\alpha - (s-1)^\alpha]/2,\ s = 1,\ \ldots,\ r-1.$$

An ES-1061 was used to compute $T_{mk} = T_k(t_m)$ from (2.5); Figs 1-6 show the results. By analogy with [2, 12], we take the criterion defining the critical time and thus the critical load as the condition that the deflection should not exceed the radius of inertia of the cross section.

Figures 1 and 2 show results for $S^* = 0, 1$, $T_{0k} = 10^{-3}$, f = 0, $\gamma = 0$; the abscissa is t^*, which is the ratio of the variable compressive force to the Euler load, while the ordinate is the dimensionless deflection sagitta T_k. Curves 1-3 correspond to k = 1, 2, and 3, while the solid and dashed lines in Fig. 1 correspond to the elastic case (A = 0) and the viscoelastic case (A = 0.05). By analogy with the elastic case [2], there is a marked increase in the deflection when the rod bends in two half-waves (k = 2). To judge from curve 2, deflection equal to the radius of inertia of the cross section is attained with $P_{cr} = 9.06P_e$, while in the elastic case $P_{cr} = 9.6P_e$, which shows that the critical load is reduced when the viscoelastic parameters are introduced.

Calculations were also performed for the viscosity coefficients A = 0.03; 0.08; 0.1 (Fig. 2); in these cases, the dynamic coefficient K_d is the ratio of the dynamic critical load to the static (Euler) one, the values being respectively 9.3, 8.76, and 8.58. The critical load is reduced as the viscosity coefficient increases.

Figure 3 was constructed with $T_{0k} = 10^{-3}$, f = 0, $\gamma = 0$ for various S^*; we give curves for the k for which the increase in the deflection is rapid. K_d increases as S^* decreases, and for $S^* = 1$, the critical number of half-waves k is 2, in contrast to the elastic case, where k = 1.

Figure 4 indicates the effects of an initial deflection in dynamic loading; we give the critical cruves T_k for $S^* = 0,1$, f = 0, $\gamma = 0$, k = 2 for successively decreasing T_{0k} in the range from 10^{-1} to 10^{-4}. For initial deflection sagittas $T_{0k} = 10^{-1}$, 10^{-2}, 10^{-3}, 10^{-4} (lines 1-4), K_d = 5.1, 7.2, 9.1, 10.5.

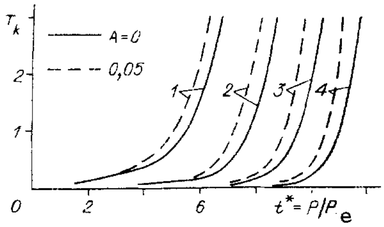

Fig. 4

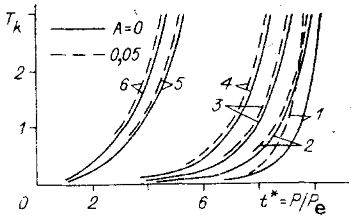

Fig. 5

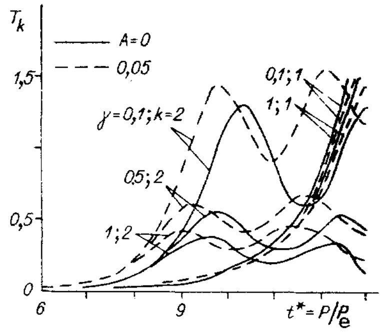

Fig. 6

We examined the effect of an additional static transverse load on the behavior (Fig. 5); k = 1 for $T_{0k} = 10^{-3}$, $\gamma = 0$, $S^* = 0, 1$ and $f \neq 0$, while k = 2 for f = 0. The transverse load tends to reduce the occurrence of higher instability forms. The K_d for f = 0, 0.025, 0.05, 0.1, 1, 1.5 (lines 1-6) are correspondingly 9.06, 8.7, 7.68, 6.96, 3.66, 3.12.

We examined how physical nonlinearity affected the behavior. Figure 6 ($T_{0k} = 10^{-3}$, $S^* = 0, 1$, f = 0) gives T_k curves for $\gamma = 0.1, 0.5, 1$, with correspondingly $K_d = 9.2, 12.2, 12.4$. For $\gamma \leq 0.1$, the solutions in the linear and nonlinear cases are similar, while for $\gamma > 0.1$, they differ substantially, the difference being about 30%, for example, for $\gamma = 0.5$. The critical loads and times are increased by incorporating the nonlinearity in the material.

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